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Models

Emanuel Derman

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Model Airplanes

When I was in grade school, we used to build model airplanes from kits. The frame was made of precut pieces of balsa wood, each having been carefully pinned, according to the plans, along a preprinted arc to obtain the appropriate curvature and then cemented, piece to piece, with airplane glue. The fuselage, made of tissue paper, was glued to the balsa frame, trimmed, dampened with water to shrink it taut, and, finally, when dry, lacquered and painted to make it stiff and realistic. The engine was just a long rubber band that ran the internal length of the fuselage, from propeller block at the nose to the tail, wound up by rotating the propeller many times and then let loose to unwind for a flight of perhaps 10 seconds at best. An especially ambitious model builder would follow the instructions very carefully—sanding off, for example, any excess glue on the frame so as to leave no imperfections whatsoever.

What was “model” about model airplanes? The Zippy model airplane that I remember building was smaller than a real Zippy (I assumed that an actual Zippy airplane existed somewhere in the world of real airplanes). It was lighter than a real airplane and made of different materials. But it did capture two essential features of the putatively real Zippy: appearance and flight. The model looked a lot like an airplane, and it could fly, if only briefly.

Nevertheless, the model was not the thing itself. It was a model Zippy. It lacked seats, ailerons, and proper windows and doors, among many other real-life details. Which features are important depends on the model user. In my case, had I been three or four years old, crudely shaped wings, a body, and a throaty airplane engine noise might have satisfied me. When I was about 10 years old, appearance and flight sufficed. When I was a few years older, I would have wanted a combustion engine and radio control. But none of these *model* Zippys, however complex, would have been the real thing.

What constrains the construction of a model Zippy?

1. The user and his or her needs: What aspects of the real airplane and its features is the user most interested in simulating, testing, or playing and tinkering with? An engineer needs a model different from that of a child.
2. Engineering and construction: How does one put together a reliable and effective model, with the key features as accurate as possible?
3. Science: Even though the Wright brothers probably did not know the partial differential equations of fluid flow, heavier-than-air flight was built on the science of mechanics and aerodynamics, Newton’s laws, and the Navier–Stokes equations.

Emanuel Derman is a professor at Columbia University and a principal at Prisma Capital Partners, New York City.

Models in Physics

Scientific models are different from models of airplanes. Resemblance is not enough. Scientific models aim at divination—foretelling the future and controlling it—and physicists use two different approaches in creating such models.

Fundamental Models. The first approach is to build what physicists call a fundamental model, which describes the *dynamics* behind events in the real world. A fundamental model consists of a system of principles, usually formulated mathematically, that is used to draw *causal* inferences about future behavior. *Dynamics* and *causality* are a fundamental model's essential characteristics. A fundamental model, particularly a successful one, is more of a theory than a model. To put it a little pedantically, fundamental models proclaim, "These are the laws of the universe." They describe the dynamics in God's terms; they seek to state eternal truths, like Moses coming down from the mountain.

A fundamental model that all physicists are familiar with is Newton's laws of mechanics and gravitation:

$$F = ma,$$

and

$$F = \frac{GMm}{r^2}.$$

The first equation is Newton's second law of motion and states that force produces acceleration. The second equation is the inverse-square law of universal gravitation and describes how mass causes gravitational force. Newton's laws are laws of cause and effect. Newton's theory isolates the appropriate variables and specifies a causal relationship between them.

The gap between a successful theory and the part of the universe it describes is virtually nonexistent: The theory is the universe, not a model of the universe; the universe is the theory.

Phenomenological Models. The second type of model is what physicists call a phenomenological model. Like fundamental models, phenomenological models are used to make predictions, but they do not state absolute principles; instead, they make pragmatic analogies between things one would like to understand and things one already understands from fundamental models. The analogies can be descriptive and useful, but analogies are self-limiting and often have a toylike quality. In physics, one does not delude oneself into thinking of analogies as truth.

Phenomenological models do not say, "This is a law." Instead, they say, "Approximately, you can think of this part of the world as being a lot like this other kind of thing that you already understand more deeply." Phenomenological models describe the world in man's language rather than God's.

A good example is the liquid drop model of the nucleus, which allows us to think of an atomic nucleus as behaving much like an oscillating drop of fluid even though we know that a nucleus is composed of individual protons and neutrons. Calibrating the liquid drop's parameters to match the known properties of the nucleus, we can then use the model to compute and predict values of other, unmeasured properties.

The gap between a successful phenomenological model and the part of the universe it describes is quite large. A phenomenological model is an approximation—a realistic-looking wax apple, Parrhasius's painted curtain that fooled his fellow artist, a wonderful resemblance—but not the thing itself.

Models in Finance

What is the point of a model in finance?

Only a little experience is needed to see that the point of a model in finance is not the same as the point of a model in physics or applied mathematics. Consider this simple but prototypical financial model: How do we estimate the price of a seven-room apartment on Park Avenue if someone tells us the market price of a typical two-room apartment in Battery Park City? Most likely, we figure out the price per square foot of the two-room apartment. Then we multiply by the square footage of the Park Avenue apartment. Finally, we make some rule-of-thumb corrections for location, park views, light, facilities, and so on.

The model's critical parameter is the implied price per square foot. We calibrate the model to Battery Park City. Then, we use the model to interpolate or extrapolate to Park Avenue. The price per square foot is *implied* by the market price of the typical Battery Park City apartment. The price per square foot is not the construction price per square foot, because other variables—exposure, quality of construction, neighborhood—are subsumed in the price per square foot.

The Aim of Financial Models. The way property markets use implied price per square foot illustrates the functions of financial models generally.

- *Models are used to rank securities by value.* Implied price per square foot can be used to rank and compare many similar but not identical apartments. As stated above, apartments have many features that affect their value. Implied price per square foot provides a simple, one-dimensional scale on which to begin ranking apartments by value. The single number given by implied price per square foot does not truly reflect the value of the apartment; it provides a starting point, after which other, qualitative factors must be taken into account.

Similarly, yield to maturity for bonds allows us to compare the values of many similar but not identical bonds, each with a different coupon and/or maturity, by mapping their yields onto a linear scale. We can do the same thing with P/E for stocks and option-adjusted spread (OAS) for mortgages or callable bonds. All of these metrics reduce a multidimensional problem to a one-dimensional problem. The volatility of options implied by the Black–Scholes model provides a similar way to collapse multiqualitied instruments (characterized by strike, expiration, underlier, etc.) onto a single value scale and make pragmatic modifications to it.

- *Models are used to interpolate or extrapolate from liquid prices to illiquid prices.* In finance, models are used less for divination than for interpolation or extrapolation from the known dollar prices of liquid securities to the unknown dollar values of illiquid securities—in our example, from the Battery Park City price to the Park Avenue value. Most financial models do not predict the future; instead, they allow us to compare different prices in the present. Similarly, OAS is used to interpolate from relatively liquid bonds to less liquid ones. Correspondingly, the Black–Scholes model proceeds from a known stock price and a riskless bond price to the unknown price of a hybrid security—an option—much in the same way one estimates the value of fruit salad from its constituent fruits or, inversely, the way one estimates the price of one fruit from the prices of the other fruits in the salad.

None of these metrics is strictly accurate, but they all provide immensely helpful ways to begin to estimate value.

- *Models transform intuitive linear quantities into nonlinear dollar values.* In physics, a theory predicts the future. In finance, a model translates intuition into dollar values. The apartment-value model transforms price per square foot into the dollar value of the apartment. Starting from price per square foot (or per room) is intuitively easy because it captures much of the variability of apartment prices. Similarly, P/E describes much of the variability of share prices. Developing intuition about yield to maturity, option-adjusted spread, default probability, or return volatility is harder than thinking about price per square foot. Nevertheless, all of these parameters are clearly related to value and easier to think about than dollar value itself. They are intuitively graspable, and the more sophisticated one becomes, the richer one's intuition becomes. Models are developed by leapfrogging from a simple, intuitive mental concept (e.g., volatility) to the mathematics that describes it (e.g., geometric Brownian motion, the Black–Scholes model), to a richer mental concept (e.g., the volatility smile), to experience-based intuition about it, and, finally, to a model (e.g., a stochastic volatility model) that incorporates the new concept.

In contrast to both fundamental and phenomenological models, the gap between a successful financial model and the correct value is nearly indefinable because fair value is finance's fata morgana, undefined by prices, which themselves are not stationary. So, model success is temporary at best. If fair value were precisely calculable, markets would not exist.

The qualities of models in different fields are summarized below.

Field	Model Aims
Physics	Reproduction, divination
Hobbyists	Resemblance
Finance	Ranking, interpolation, intuition

The Foundations of Financial Engineering

Science—for example, mechanics, electrodynamics, or molecular biology—seeks to discover the fundamental principles that describe the world and is thus usually reductive. Engineering is about constructively using those principles for a purpose.

Mechanical engineering is concerned with building devices based on the principles of mechanics (Newton's laws), suitably combined with empirical rules about complex forces (e.g., friction) that are too difficult to derive from first principles.

Electrical engineering is the study of how to create useful electrical devices based on Maxwell's equations and solid-state physics. Bioengineering is the art of building prosthetics and other biologically active devices based on the principles of biochemistry, physiology, and molecular biology.

What about financial modeling, financial engineering, and quantitative finance? In a logically consistent world, financial engineering, layered above a base of solid financial science, would be the study of how to create functional financial devices—convertible bonds, warrants, credit default swaptions—that perform in desired ways not only at expiration but also throughout their lifetimes. Financial science is the study of the fundamental laws of financial objects: stocks, interest rates, or whatever else a theory uses as its "atomic" constituents. Here, unfortunately, lie dragons.

Brownian motion, the underpinning of much of quantitative finance, is indeed science, but it is accurate only for small particles bumped around by invisible atoms. For stocks, the standard theory of geometric Brownian motion is an idealization that captures some of the essential features of price uncertainty but is not a very good description of the detailed characteristics of stocks' price distributions. Markets are both plagued and blessed with anomalies that disagree with standard and nonstandard theories. Thus, although we financial engineers are rich in techniques (stochastic calculus, optimization, the Hamilton–Jacobi–Bellman equation, etc.), we do not yet have the right laws of science to exploit.

What solid laws and concepts do we have for building our ranking and translation models? In truth, only one.

The One Law of Financial Modeling. According to legend, Hillel, a famous Jewish sage, was asked to recite the essence of God's laws while standing on one leg. "Do not do unto others as you would not have them do unto you," he is supposed to have said. "All the rest is commentary. Go and learn."

Similarly, we can summarize the essence of quantitative finance on one leg: If you want to know the value of a security, use the known price of another security that is as similar as possible to the first security. All the rest is modeling. Go and build.

"Security" refers not only to a single security but also to a portfolio of securities. The wonderful thing about this law—valuation by analogy—is that, in contrast to almost everything else in economics, it dispenses with utility functions, the unobservable hidden variables whose ghostly presence permeates most of faux-quantitative economic theory. Financial economists refer to this essential principle as the law of one price or the principle of no riskless arbitrage, which states that any two securities with identical estimated future payoffs, no matter how the future turns out, should have identical current prices.

The law of one price—this valuation by analogy—is the only genuine law in quantitative finance, and it is not a law of nature.[1] It is a general reflection on the practices of human beings—who, when they have enough time and enough information, will grab a bargain when they see one. The law of one price usually holds over the long run in well-oiled markets with enough savvy participants, but short-lived and even long-lived and persistent exceptions can always be found.

How do we use the law of one price to determine value? If we want to estimate the unknown value of a target security, we must find some other replicating portfolio—a collection of liquid securities that has the same estimated future payoffs as the target no matter how the future turns out. The target's value is simply the value of the replicating portfolio.

Where do models come in? One needs a model to show that the target and the replicating portfolio have identical estimated future payoffs under all circumstances. To demonstrate payoff identity, we must (1) specify what we mean by "all circumstances" for each security and (2) find a strategy for creating a replicating portfolio that in each future scenario or circumstance will have payoffs identical to those of the target. That is what the Black–Scholes option pricing model does: It tells us exactly how to replicate or manufacture fruit salad (an option) out of fruit (stocks and bonds). The appropriate price should be the cost of manufacture.

The tricky part in building these models is specifying what we mean by "all circumstances." In the Black–Scholes model, all circumstances means a future in which stock returns are normally distributed and stock prices move continuously. Unfortunately, real stock prices do not behave that way. Trying to specify all circumstances brings to mind the 1967 movie *Bedazzled*, starring Peter Cook and Dudley Moore. In this retelling of the German legend of Faust, Dudley Moore plays a short-order cook at a Wimpy's chain restaurant in London who sells his soul to the devil in exchange for seven chances to specify the circumstances under which he can achieve his romantic aims with the Wimpy's waitress he desires. Each time that the devil asks him to specify the romantic scenarios under which he believes he will succeed, he cannot get them quite specific enough. He says he wants to be alone with the waitress in a beautiful place where they are both in love with each other. He gets what he wants—with a snap of the devil's fingers, he and his beloved are instantly transported to a country estate where he is a guest of the owner, her husband, whom her principles will not allow her to betray. In the final episode, he wishes for them to be alone together and in love in a quiet place where no one will bother them. He gets his wish: The devil makes them both nuns in a convent where everyone has taken a vow of silence. This difficulty is the same difficulty we have when specifying future scenarios in financial models—like the devil, markets always outwit us eventually. Even if markets are not strictly random, their vagaries are too rich to capture in a few sentences or equations.

Model Risk

Risk is future uncertainty. A coin flip is risky. We know the current state of the coin but not its future state. We can, however, perform an infinite number of mental flips and reliably calculate the probability distribution of heads and tails, which will match a physical coin's probability distribution to the extent that the coin is separable from its surroundings and uninfluenced by them. In that sense, a liquid stock price is risky. We know the current price (more or less) and have no idea about the direction of its future change. But we cannot perform an infinite number of mental stock price moves with any reliability; the stock, the market, and the world are not clearly separable and they do influence each other, so the probability distribution of stock prices cannot be accurately known (and may not be time-invariant). The history of the world does not affect a coin flip. The history of the world does have a bearing on the next change in a stock's price. The risk of a stock price change is qualitatively different from the risk of a coin flip.

Financial models interpolate from liquid to illiquid prices by analogy and must necessarily change over time as the economic environment changes or as market participants become more sophisticated. The Black–Scholes model, for example, used to be regarded as adequate for valuing exotic options before the market crash of 1987, but now it is often replaced by a range of extended models that incorporate local volatility, stochastic volatility, or jumps. One cannot know the correct current model, let alone the future one, so the correct model is uncertain not only in the future but also in the present (Derman 2001). The term "risk," therefore, inaccurately describes the indeterminate nature of financial models. If we want to describe this state of ignorance as risk, then we must not forget that it is shorthand for uncertainty, for something much vaguer than probabilistic risk. No ensemble of models exists in which each model has a known probability of being right.

Conclusion

The greatest danger in financial modeling is the age-old sin of idolatry. Financial markets are alive, but a model is a limited, human work of art. Although a model may be entrancing, we will not be able to breathe life into it, no matter how hard we try. To confuse the model with the world is to embrace a future disaster driven by the belief that humans obey mathematical rules.

Thus, financial modelers must compromise by deciding what small part of the financial world is of greatest current interest, focusing on its key features, and making a mock-up of only those features. A model cannot include everything. If one is interested in everything, one is interested in too much. A successful financial model has limited scope. We must work with simple analogies. In the end, we are trying to rank complex objects on a low-dimensional scale. In physics, a theory of everything may one day exist; in finance and the social sciences, one is lucky to find a usable theory of anything.

Models are best regarded as a collection of parallel, inanimate "thought universes" to explore. Each universe should be internally consistent, but the financial/human world, unlike the world of matter, is vastly more complex and vivacious than any model we could ever make of it. We are always trying to shoehorn the real world into one of our models to see how usefully the model approximates the key features that interest us.

The right way to engage with a model is to be like a reader of fiction—to suspend disbelief and then push ahead with the model as far as possible. The story of the theory of options valuation, the best model economics currently offers, is the story of a platonically simple theory taken more seriously than it deserves and then used extravagantly, with hubris, as a crutch to human thinking. "If the fool would persist in his folly he would become wise," wrote William Blake in *The Marriage of Heaven and Hell*. That is what options markets have done with options theory.

A little hubris can be a good thing. But catastrophe strikes when hubris evolves into idolatry. Somewhere between these two extremes, a little north of common sense but still south of idolatry, lies the wise use of conceptual models.

This article qualifies for 0.5 CE credit.

Notes

1. The time value of money, the benefits of diversification, and the value of the right to choose are other useful principles. I thank Marcos Carreira for pointing them out to me.

References

Derman, E. 2001. "Markets and Models." *Risk*, vol. 14, no. 7 (July):48–50.